# Robustly Regulating a Monopolist's Access to Consumer Data*

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**Abstract**

I study how to regulate a monopolist's access to consumer data when a regulator faces non-Bayesian uncertainty about how the monopolist will exploit the consumer's information to segment the market and set prices. I fully characterize all worst-case optimal policies when the regulator maximizes consumer surplus: the regulator allows a monopolist to access data only if the monopolist cannot use the database to identify a *small group* of consumers.

**Keywords:** price discrimination, consumer data, regulation, robustness, worst-case optimality
**JEL Codes:** D42, D81, L12, L43

## 1 Introduction

The growing accessibility of consumer data enables firms to obtain unprecedented insights into consumer behavior. In markets where a monopolist can use such data to tailor prices across consumers, these insights may reveal information about consumers' willingness to pay and thereby facilitate price discrimination. While consumer data can serve many purposes, this use has important implications for welfare and consumer surplus. This raises the question: How should consumer privacy laws regulate a monopolist's access to consumer data when that data may be used for price discrimination?

To address this question, it is essential to understand how a monopolist will utilize consumer data to make pricing decisions. However, recent developments in collecting, transmitting, and analyzing consumer data—along with firms' strong incentives to protect this information—suggest that policymakers and regulators may lack insight into the specific ways data is used for price discrimination. For instance, firms may employ marketing metrics that are inaccessible to regulators, which are crucial for understanding the connection between data and consumers' willingness to pay. Such insights then enable firms to segment the market and set prices accordingly. Additionally, firms can match consumer data with other information, including past purchase decisions—capabilities that regulators do not possess—and use this information for price discrimination. More broadly, the Council of Economic Advisors report on big data and price discrimination (CEA, 2015) states:

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> "Although a few studies have tried to detect differential pricing online, current knowledge is mainly anecdotal. Companies naturally protect information about pricing strategies for competitive reasons, and perhaps also for fear of a customer backlash."

In this paper, I explore this question by analyzing a scenario in which a regulator concerned about consumer surplus has the authority to control which data a monopolist can access, yet faces non-Bayesian uncertainty regarding how the monopolist will exploit this consumer information to segment the market and set prices. More formally, I represent this uncertainty by assuming that the regulator lacks knowledge of the correlation structure between the data and consumers' willingness to pay.

I consider a model in which a monopolist (he) sells a good to a unit mass of consumers. Each consumer is privately informed about their valuation for the good, which takes finitely many values. The monopolist can acquire additional information about consumers through what I call a *database*. A database specifies the correlation between consumers' valuations and a *covariate*, which assigns a label from a finite set to each consumer. This covariate may capture characteristics such as income, gender, or race. The database is valuable because it allows the monopolist to segment the market and engage in third-degree price discrimination.

In contrast, a regulator (she) that aims to maximize consumer surplus implements a policy that controls which covariates the monopolist may access. She knows the marginal distributions of valuations and labels, but not their joint distribution; that is, she lacks access to the underlying database. To address this uncertainty, she evaluates each covariate based on its *worst-case* consumer surplus across all joint distributions consistent with the marginals. A policy is *worst-case optimal* if every covariate authorized for access by the regulator yields the maximum worst-case consumer surplus.

My main result, Theorem 1, fully characterizes the set of covariates that attain the maximum worst-case consumer surplus, enabling the identification of all worst-case optimal policies available to the regulator. The theorem establishes a simple necessary and sufficient condition: a covariate yields the maximum worst-case consumer surplus if and only if the proportion of consumers assigned to every label exceeds a uniform lower bound, which I characterize analytically. Equivalently, a policy is worst-case optimal if and only if it does not allow the monopolist to identify a sufficiently small group of consumers. This result provides an economic rationale for why some consumer privacy laws restrict access to data that can identify small or minority groups. For example, the Equal Credit Opportunity Act (ECOA) prohibits credit companies from using information that reveals consumers' race, ethnicity, or religion when assessing creditworthiness.

The intuition is as follows. For any covariate, the joint distribution where labels and valuations are independent is always consistent with the marginals. This scenario reverts to uniform pricing and therefore generates the consumer surplus associated with the uniform monopoly price. Since such a joint distribution is always feasible, this consumer surplus is an upper bound on the worst-case consumer surplus. Moreover, a trivial policy that prohibits access to consumer data always generates this benchmark level of consumer surplus. Thus, a covariate attains the maximum worst-case consumer surplus if and only if it guarantees a consumer surplus at least as large as the uniform-pricing benchmark for every plausible joint distribution. The key insight of Theorem 1 is that identifying a sufficiently small group of consumers allows for joint distributions in which that group is disproportionately composed of high-valuation consumers. In such cases, the monopolist can profitably charge prices above the uniform monopoly price to that group, reducing consumer surplus below the benchmark level. Conversely, when every label contains a sufficiently large fraction of consumers, such adverse segmentations are impossible.

After characterizing all worst-case optimal policies available to the regulator, I consider a refinement in the spirit of Börgers (2017) and ask the following question: From the set of covariates that attain the maximum worst-case consumer surplus, which ones are undominated? That is, for which covariates is there no alternative covariate whose set of attainable consumer surplus levels uniformly dominates its own?

Theorem 2 identifies the covariates that yield the maximum worst-case consumer surplus and are also undominated. These covariates not only consistently achieve a consumer surplus weakly higher than that obtained through uniform pricing but can also yield a consumer surplus strictly superior to that attained via the uniform monopoly price for certain feasible joint distributions of valuations and labels. Theorem 2 establishes both the sufficient and necessary conditions for a covariate to exhibit this property. In addition to satisfying the uniform lower bound set by Theorem 1, the proportion of consumers assigned to one of the labels is below an upper bound, which I also characterize.

Finally, Theorem 1 and 2 also generalize to the case where the regulator's objective is to maximize a weighted average of consumer and producer surplus, where the weight assigned to consumer surplus is weakly higher than the weight assigned to producer surplus.

The remainder of this paper is organized as follows. Section 1.1 reviews the related literature. Section 2 introduces the model. Section 3 shows the main results of this paper. Section 4 extends the model and results to alternative objectives for the regulator, and Section 5 presents concluding remarks.

### 1.1 Related Literature

This paper contributes to the literature on data markets recently surveyed by Bergemann and Bonatti (2019). Within this literature, I am mainly motivated by the insights of Bergemann et al. (2015). In this work, the authors characterize all combinations of consumer and producer surplus that can emerge from third-degree price discrimination. This characterization shows that market outcomes can be quite heterogeneous when the monopolist has additional information about the market beyond the prior distribution. This paper then asks: If a regulator prioritizes consumer surplus, what additional information about the market should she permit the monopolist to access?

Ali et al. (2022), Pram (2021), Vaidya (2023) and Argenziano and Bonatti (2021) also analyze the regulation of data markets. However, my paper differs from these studies in two aspects. First, prior studies approach regulation from a positive perspective, analyzing the welfare effects of granting consumers control over their data. In contrast, my paper adopts a normative approach. Specifically, it explores a different question: If a regulator can enact any policy governing firms' access to data, what form should such policy take? Second, in prior studies, every player has complete knowledge of the relevant environment. In contrast, in my setting, the regulator possesses non-Bayesian uncertainty concerning the correlation between data and willingness to pay.

Another relevant strand of the literature examines mechanism and information design in monopoly settings under worst-case or worst-regret objectives. Du (2018) examines an auction for selling a common-value good, finding that the minimum revenue across all information that bidders might hold about the good's value converges to the full surplus as the number of bidders increases. Carroll (2017) explores a setting where a monopolist sells multiple goods to a buyer. The monopolist knows the marginal distribution of the buyer's valuation for each good but is uncertain about the joint distribution. Concurrently, Che and Zhong (2021) and Deb and Roesler (2021) also analyze scenarios in which a monopolist sells multiple goods to a buyer. In Che and Zhong (2021), the monopolist knows only that the distribution of the buyer's valuations lies within a certain set, without assigning a prior over it. In Deb and Roesler (2021), the monopolist faces non-Bayesian uncertainty about the information the buyer may acquire regarding her valuations.

In all the setups above, it is the monopolist who faces non-Bayesian uncertainty. In contrast, my paper centers on a regulator who faces non-Bayesian uncertainty about the information available to the monopolist. Related to this idea, Guo and Shmaya (2019) considers a regulator that has non-Bayesian uncertainty about the demand function and cost function faced by the monopolist.

Lastly, as the comment made by Börgers (2017) on the work by Chung and Ely (2007) and the work by Dworczak and Pavan (2022), this paper explores the dominance order among policies that achieve the maximum worst-case payoff.

# 2 Model

A monopolist (he) sells a good to a unit mass of consumers, each of whom demands one unit. The monopolist faces constant marginal costs of production that I normalize to zero. The consumers privately observe their *willingness to pay* (*valuation*) for the good, which can take $K$ possible values, $V \equiv \{v_1, ..., v_k, ..., v_K\}$, with $v_k \in \mathbb{R}_+$; without loss of generality assume: $0 < v_1 < \cdots < v_K$.

A *market* $x$ is a distribution over the $K$ valuations, with the set of all markets being

$$\Delta(V) \equiv \left\{ x \in \mathbb{R}_+^K \;\middle|\; \sum_{k=1}^{K} x_k = 1 \right\},$$

where $x_k$ is the proportion of consumers who have valuation $v_k$. Furthermore, I denote by $\bar{x}_k$ the proportion of consumers who have a valuation greater than or equal to $v_k$. That is, $\bar{x}_k = \sum_{j=k}^{K} x_j$. Additionally, I say that the price $v_k$ is *optimal* for market $x$ if the expected revenue from price $v_k$ satisfies $v_k \bar{x}_k \geq v_i \bar{x}_i$ for all $i \in \{1, \ldots, K\}$.

Throughout the analysis, I fix and denote by $x^* \in \Delta(V)$ the market faced by the monopolist. Let $v_{i^*}$ be the highest price among all optimal prices for market $x^*$, where

$$v_{i^*} = \max \left\{ \arg\max_{v_k \in V} \; v_k \bar{x}_k^* \right\}.$$

I will refer to $v_{i^*}$ as the *uniform monopoly price*. Given this uniform monopoly price, the corresponding consumer surplus is given by

$$u^* = \sum_{j=i^*}^{K} (v_j - v_{i^*}) x_j^*.$$

To avoid trivialities in the analysis, assume that $u^* > 0$[1]. This implies that there is a valuation $v_k$ higher than the uniform monopoly price $v_{i^*}$, and the proportion of consumers that have valuation $v_k$ is strictly positive. That is, $x_k^* > 0$.

## 2.1 Data

Formally, a *database* is a joint distribution $d \in \Delta(V \times S)$ over the set of valuations $V$ and a finite set of labels $S$, where $d_{k,s}$ denotes the share of consumers with valuation $v_k$ and label $s$. Let $f \in \Delta(S)$

[1] As I will show later in Lemma 1, a covariate yields the maximum worst-case consumer surplus if and only if it achieves a consumer surplus weakly greater than $u^*$ for all plausible conditional distributions of valuations given labels. If $u^* = 0$, then any covariate yields the maximum worst-case consumer surplus. To avoid this trivial case, I assume that $u^* > 0$.

denote the marginal distribution over labels. Without loss of generality, assume full support: $f_s > 0$ for all $s \in S$. I refer to $f$ as a *covariate distribution*, and to the pair $(S, f)$ as a *covariate*.

Since the specific labels in $S$ play no intrinsic role in the analysis, we normalize by letting $S \equiv \{1, 2, \ldots, n\}$ and identify each covariate with its covariate distribution $f$. The set of all possible covariate distributions—marginal distributions over finite label sets with full support—is given by

$$F \equiv \bigcup_{n \in \mathbb{N}} \left\{ f \in \mathbb{R}^n_{++} \;\middle|\; \sum_{s=1}^{n} f_s = 1 \right\}.$$

A covariate may represent either a single consumer characteristic or a collection of characteristics. For example, suppose consumers are described by finitely many characteristics such as gender, income, age, and zip code. Then the label space can be written as a finite product set $S = S_1 \times \cdots \times S_k$, where each $S_i$ corresponds to one characteristic. A label $s \in S$ records the joint realization of these characteristics, and $f \in \Delta(S)$ is the corresponding covariate distribution. Since $S$ is finite, this covariate distribution belongs to $F$. Thus, the set $F$ already accommodates covariate distributions that encode an arbitrary finite number of underlying consumer characteristics.

A special case arises when $n = 1$, yielding a unique trivial covariate distribution with singleton support, denoted by $f^{\emptyset}$. A database with covariate distribution $f^{\emptyset}$ corresponds to a setting in which no information is available—that is, all consumers share the same label.

## 2.2 Value of Data

Given a database $d$ with covariate distribution $f$, define $\sigma(\cdot \mid s) \in \Delta(V)$ as the conditional distribution over valuations given label $s$. Specifically, $\sigma(k \mid s) = \frac{d_{k,s}}{f_s}$ represents the share of consumers with valuation $v_k$ conditional on label $s$. I refer to $\sigma(\cdot \mid s)$ as *segment* $s$, and I denote the full collection of these conditional distributions by $\sigma \equiv \{\sigma(\cdot \mid s)\}_{s=1}^{n}$, calling this collection a *segmentation*.

Note that since $d$ must be consistent with the marginals $x^*$ and $f$, the segmentation decomposes the aggregate market in the sense that

$$\sum_{s=1}^{n} f_s \sigma(k \mid s) = \sum_{s=1}^{n} d_{k,s} = x^*_k, \quad \text{for all } k \in \{1, \ldots, K\}.$$

If the monopolist has access to covariate distribution $f$, he also has access to the corresponding database $d$. That is, the monopolist knows the joint distribution of valuations and labels. As a result, he will set profit-maximizing prices within each segment. In segment $s$, the chosen price is $v_{i^*_s}$, where

$$v_{i^*_s} = \max \left\{ \arg\max_{v_k \in V} \; v_k \sum_{j=k}^{K} \sigma(j \mid s) \right\}. \tag{1}$$

If multiple prices yield the same profit, ties are broken against the consumer.

Given these prices, the associated consumer surplus is given by

$$\underline{u}(\sigma, f) = \sum_{s=1}^{n} f_s \sum_{j=i^*_s}^{K} (v_j - v_{i^*_s}) \sigma(j \mid s).$$

### 2.3 Regulation

A regulator (she), concerned with consumer surplus, has the authority to control which covariate distribution the monopolist may access. Formally, the regulator selects a policy $\mathcal{F} \subseteq F$, and the monopolist can access any covariate distribution from this set.

Recall that if the monopolist has access to $f$, he also has access to the corresponding database $d$. In contrast, the regulator is assumed to know only the market distribution $x^*$ and faces non-Bayesian uncertainty about the underlying joint distribution $d$. To address this, she evaluates each covariate distribution based on its *worst-case consumer surplus*, defined as the minimum attainable surplus across all joint distributions consistent with $x^*$ and $f$. For any $f$, define the set of *feasible segmentations* as

$$\Sigma(f) \equiv \left\{ \sigma = \{\sigma(\cdot \mid s)\}_{s=1}^{n} \text{ with each } \sigma(\cdot \mid s) \in \Delta(V) \;\middle|\; \sum_{s=1}^{n} f_s \sigma(\cdot \mid s) = x^* \right\}.$$

Then, the regulator evaluates $f$ by

$$\inf_{\sigma \in \Sigma(f)} \underline{u}(\sigma, f).$$

A policy $\mathcal{F}$ is said to be *worst-case optimal* if every $f \in \mathcal{F}$ yields the maximum worst-case consumer surplus. That is, $\mathcal{F} \subseteq WC$, where

$$WC \equiv \arg\max_{f \in F} \inf_{\sigma \in \Sigma(f)} \underline{u}(\sigma, f).$$

The following Lemma gives a simple expression for $WC$ and shows that it is non-empty.

**Lemma 1.** *Let $F_1 = \{f \in F \mid \underline{u}(\sigma, f) \geq u^* \; \forall \, \sigma \in \Sigma(f)\}$ be the set of covariate distributions that yield a consumer surplus greater than or equal to $u^*$ for all feasible segmentations. Then $f^{\emptyset} \in F_1$ and $WC \equiv F_1$.*

*Proof.* For any $f \in F$, $\sigma(\cdot|s) = x^*$, for all $s$, belongs to $\Sigma(f)$. In other words, the joint distribution where labels and valuations are independent is always consistent with the marginals. In this case, charging the uniform monopoly price in every segment is optimal; therefore, consumer surplus equals $u^*$. Moreover, $u^*$ is an upper bound on the worst-case consumer surplus. Additionally, for $f^{\emptyset} \in F$, the only feasible consumer surplus is $u^*$. Hence, $f^{\emptyset} \in WC$, and $f \in WC$ if and only if $\underline{u}(\sigma, f) \geq u^*$ for all $\sigma \in \Sigma(f)$. □

Lemma 1 reduces the regulator's problem to a benchmark comparison. Any covariate distribution that guarantees the consumer surplus obtained under the uniform monopoly price for every feasible segmentation also achieves the maximum worst-case consumer surplus. Consequently, characterizing the set $WC$ is equivalent to identifying the covariate distributions with this property.

## 3 Results

I start by presenting the main result of this paper. This result fully characterizes the set of covariates that yield the maximum worst-case consumer surplus.

**Theorem 1.** *A covariate distribution $f$ yields a consumer surplus greater than or equal to $u^*$ for all feasible segmentations $\sigma \in \Sigma(f)$ if and only if $f_s > \underline{\lambda}$ for every label $s$, where*

$$\underline{\lambda} = \min_{v_j \leq v_{i^*}} \frac{\max_{v_i > v_{i^*}} \; v_i \bar{x}_i^*}{v_j} + 1 - \bar{x}_j^*.$$

*If direction of Theorem 1.* Suppose that $f_s > \underline{\lambda}$ for all $s$. Towards a contradiction, assume that there is a feasible segmentation $\sigma \in \Sigma(f)$ such that $\underline{u}(\sigma, f) < u^*$. Then a segment $s$ must exist where the monopolist sets a price higher than $v_{i^*}$. Denote such a price by $v_{k_s}$. Since $v_{k_s}$ maximizes the monopolist's profits in segment $s$, for all $v_j < v_{k_s}$ it follows that

$$\underbrace{\frac{1}{v_j}\left( v_{k_s} \sum_{k=k_s}^{K} \sigma(k|s) - v_j \sum_{k=j}^{K} \sigma(k|s) \right)}_{\text{Profits from price } v_{k_s} \; - \text{ Profits from price } v_j} = \frac{v_{k_s}}{v_j} \sum_{k=k_s}^{K} \sigma(k|s) + \sum_{k=1}^{j-1} \sigma(k|s) - 1 \geq 0. \tag{2}$$

Furthermore, since $\sigma \in \Sigma(f)$, $\sigma(\cdot|s) f_s \leq x^*$. Then, if I multiply both sides of inequality 2 by $f_s$ I get that

$$\frac{v_{k_s}}{v_j} \bar{x}_{k_s}^* + 1 - \bar{x}_j^* = \frac{v_{k_s}}{v_j} \sum_{k=k_s}^{K} x_k^* + \sum_{k=1}^{j-1} x_k^* \geq \left[ \frac{v_{k_s}}{v_j} \sum_{k=k_s}^{K} \sigma(k|s) + \sum_{k=1}^{j-1} \sigma(k|s) \right] f_s \geq f_s. \tag{3}$$

Inequality 3 holds for some $v_{k_s} > v_{i^*}$ and all $v_j \leq v_{i^*}$. Therefore,

$$f_s \leq \min_{v_j \leq v_{i^*}} \frac{\max_{v_i > v_{i^*}} \; v_i \bar{x}_i^*}{v_j} + 1 - \bar{x}_j^* = \underline{\lambda}, \tag{4}$$

which is a contradiction. □

Intuitively, the proof of the sufficiency part of Theorem 1 shows that when a covariate distribution satisfies the uniform lower bound $\underline{\lambda}$, no feasible segmentation can generate a label whose segment places enough weight on valuations above the uniform monopoly price $v_{i^*}$ to make a higher price profitable. As a result, the monopolist never finds it optimal to charge more than $v_{i^*}$ after observing any label, and consumer surplus is therefore guaranteed to be at least $u^*$.

The proof of the only if part of Theorem 1 is provided in Appendix A. It builds on the algorithm proposed by Bergemann et al. (2015), which segments $x^*$ using a "greedy" procedure that generates segments corresponding to extreme markets. An extreme market is defined as a distribution of valuations in which the monopolist is indifferent between charging any price in the support.

Specifically, I show that the total mass of consumers assigned by the algorithm to extreme markets containing valuations strictly greater than the uniform monopoly price equals $\underline{\lambda}$. I then use this result to construct a feasible segmentation that yields a consumer surplus strictly lower than $u^*$ whenever $f_s \leq \underline{\lambda}$ for some $s$. The following example illustrates how such a segmentation is constructed.

**Example 1** (Intuition behind the necessity part of Theorem 1)**.** The set of valuations is given by $V = \{1, 2, 3, 4\}$. The market faced by the monopolist is represented by the distribution $x^* = \left(\frac{2}{5}, \frac{1}{10}, \frac{2}{5}, \frac{1}{10}\right)$, where $x_1^* = x_3^* = \frac{2}{5}$ and $x_2^* = x_4^* = \frac{1}{10}$. The uniform monopoly price is 3, which yields a consumer surplus of $u^* = \frac{1}{10}$. In this case, it is straightforward to verify that the uniform lower bound is $\underline{\lambda} = \frac{2}{5}$.

Using the greedy algorithm proposed by Bergemann et al. (2015)[2], I obtain the following segmentation of $x^*$:

Table 1: Extremal segmentation for $x^*$

| Extreme markets | $v=1$ | $v=2$ | $v=3$ | $v=4$ | Mass of consumers |
|---|---|---|---|---|---|
| $x^{\{1,2,3,4\}}$ | $\frac{1}{2}$ | $\frac{1}{6}$ | $\frac{1}{12}$ | $\frac{1}{4}$ | $\frac{2}{5}$ |
| $x^{\{1,2,3\}}$ | $\frac{1}{2}$ | $\frac{1}{6}$ | $\frac{1}{3}$ | 0 | $\frac{1}{5}$ |
| $x^{\{1,3\}}$ | $\frac{2}{3}$ | 0 | $\frac{1}{3}$ | 0 | $\frac{3}{20}$ |
| $x^{\{3\}}$ | 0 | 0 | 1 | 0 | $\frac{1}{4}$ |
| $x^*$ | $\frac{2}{5}$ | $\frac{1}{10}$ | $\frac{2}{5}$ | $\frac{1}{10}$ | 1 |

The segmentation assigns a mass of $\frac{2}{5}$ of consumers to the extreme market in which the monopolist is indifferent between charging any price in the support of $x^*$. A share of $\frac{1}{5}$ is assigned to the extreme market where the monopolist is indifferent between prices in $\{1, 2, 3\}$. A mass of $\frac{3}{20}$ is allocated to the extreme market with support $\{1, 3\}$. Finally, the remaining consumers are assigned to the extreme market supported solely on valuation 3.

The algorithm proceeds greedily by repeatedly selecting the unique extreme market that makes the monopolist indifferent among all prices in the support of the current residual market and assigning to it the largest feasible mass. Each iteration exhausts at least one valuation, thereby reducing the support of the residual market. Repeating this construction eventually decomposes the original market into a finite collection of extreme markets.

The key insight from the above segmentation lies in the unique extreme market that includes valuations strictly above the uniform monopoly price, denoted as $x^{\{1,2,3,4\}}$. The mass of consumers assigned to this segment is exactly equal to the uniform lower bound, $\underline{\lambda} = \frac{2}{5}$. This leads to the following necessary condition: a covariate distribution $f$ yields a consumer surplus weakly greater than $u^*$ for all feasible segmentations only if $f_s > \frac{2}{5}$ for every label $s$.

To prove the result, I will show that if $f_s \leq \frac{2}{5}$ for some $s$, then a feasible segmentation exists that yields consumer surplus strictly below $u^*$. Define $\sigma(\cdot|s) = x^{\{1,2,3,4\}}$, and for all $s' \neq s$, define $\sigma(\cdot|s') = \frac{x^* - \sigma(\cdot|s) f_s}{1 - f_s}$. This construction satisfies three key properties: (i) Since $f_s \leq \frac{2}{5}$, the segmentation is feasible. (ii) Conditional on observing label $s$, any price in the support of $\sigma(\cdot|s)$ is optimal. Moreover, under the assumption that the monopolist breaks ties against consumers, the chosen price is 4. (iii) The price charged to consumers with any other label $s' \neq s$ is no lower than 3. To see this, suppose instead that the optimal price for some $s'$ is 1 or 2. Then, either of these prices must yield strictly higher profits than 3 for all $s' \neq s$. Furthermore, as $\sigma(\cdot|s)$ is an extreme market, 1 and 2 are an optimal price for that segment. Therefore, 1 or 2 yield strictly higher profits than 3 in the aggregate market $x^*$, which contradicts that 3 is the uniform monopoly price.

In essence, by allocating all consumers with label $s$ to the extreme market that contains valuations strictly higher than the uniform monopoly price in the support, I ensure that these consumers receive a price higher than the uniform monopoly price without counterbalancing benefits to consumers with different labels.

Theorem 1 yields four immediate corollaries that I describe next.

[2] For a formal definition of the greedy algorithm, see the proof of the only if direction of Theorem 1 in Appendix A.

**Corollary 1.** *There exist covariate distributions other than the trivial covariate distribution $f^{\emptyset}$ that achieve the maximum worst-case consumer surplus if and only if $\underline{\lambda} < \frac{1}{2}$.*

Indeed, there is no covariate distribution with finite and full support, other than $f^{\emptyset}$, that assigns a probability strictly greater than $\frac{1}{2}$ to all elements in its support. Therefore, if $\underline{\lambda} > \frac{1}{2}$, $WC \setminus \{f^{\emptyset}\}$ is empty. Conversely, if $\underline{\lambda} < \frac{1}{2}$, the covariate $f_1 = f_2 = \frac{1}{2}$ satisfies the uniform lower bound imposed by Theorem 1, and thus $f \in WC \setminus \{f^{\emptyset}\}$.

**Corollary 2.** *A covariate distribution $f$ achieves the maximum worst-case consumer surplus if and only if, for every feasible segmentation $\sigma \in \Sigma(f)$, the price charged by the monopolist in every segment $s$, denoted $v_{i_s^*}$, is weakly lower than the uniform monopoly price $v_{i^*}$.*

If $v_{i_s^*} \leq v_{i^*}$ for every $\sigma \in \Sigma(f)$, then clearly $f \in WC$. The converse follows from the if part of Theorem 1, which shows that if a price above the uniform monopoly price maximizes profits in some segment $s$, then $f_s \leq \underline{\lambda}$.

Corollary 2 shows that implementing a worst-case optimal policy guarantees that no consumer is worse off relative to the benchmark without data, since consumers never face a price above the uniform monopoly price. This property may be particularly desirable for regulators who are concerned not only with aggregate consumer surplus but also with inequality, as it ensures that the gains from allowing data do not come at the expense of any consumer.

To state the next result, recall that a covariate may represent either a single characteristic or a collection of characteristics. Let $f^1$ and $f^2$ be covariate distributions with label sets $S^1$ and $S^2$, respectively. Let $f^1 \vee f^2$ denote a joint distribution over the product set $S^1 \times S^2$ whose marginals are $f^1$ and $f^2$. For example, if the first covariate describes gender and the second covariate describes income, then $f^1 \vee f^2$ describes the joint distribution of gender and income. Since covariates in our framework may encode an arbitrary finite collection of characteristics, $f^1 \vee f^2$ is itself a feasible covariate distribution in $F$. Throughout the corollary, the joint distribution $f^1 \vee f^2$ is assumed to be known to the regulator. Theorem 1 immediately implies the following.

**Corollary 3.** *Suppose that covariate distribution $f^1 \notin WC$. Then, for any covariate distribution $f^2 \in F$, we have $f^1 \vee f^2 \notin WC$. Conversely, suppose that $f^1 \vee f^2 \in WC$. Then $f^1 \in WC$ and $f^2 \in WC$.*

Intuitively, if $f^1 \notin WC$. Then by Theorem 1, $f_s^1 < \underline{\lambda}$ for some $s \in S^1$. For any covariate $f^2$ with label set $S^2$, it follows that $(f^1 \vee f^2)_{(s,s')} \leq f_s^1 < \underline{\lambda}$ for all $s' \in S^2$.

Conversely, suppose $f^1 \vee f^2 \in WC$. Then for every $s \in S^1$ and $s' \in S^2$, we must have $(f^1 \vee f^2)_{(s,s')} > \underline{\lambda}$, which implies $f_s^1 > \underline{\lambda}$ and similarly $f_{s'}^2 > \underline{\lambda}$.

Finally, note that Theorem 1 can be extended to settings in which the regulator also seeks policies that are robust to uncertainty about the market distribution. Suppose the regulator only knows that the true market distribution lies in a set $\mathcal{X} \subset \Delta(V)$ and assigns no prior over this set. For instance, if the regulator has previously observed the uniform monopoly price, $\mathcal{X}$ may include all market distributions for which that price is optimal. Alternatively, if the regulator has observed the demand for a subset of prices $V' \subset V$, so that $\bar{x}_k$ is known for all $v_k \in V'$, then $\mathcal{X}$ consists of all markets consistent with those demand observations.

In this environment, the regulator evaluates each covariate distribution by its worst-case consumer surplus across all markets in $\mathcal{X}$ and their corresponding feasible segmentations. Formally, let $\Sigma(f, x)$ denote the set of segmentations consistent with the marginals $f$ and $x$. A policy is worst-case optimal if every admissible covariate distribution belongs to

$$WC(\mathcal{X}) \equiv \arg\max_{f \in F} \inf_{\substack{x \in \mathcal{X} \\ \sigma \in \Sigma(f,x)}} \underline{u}(\sigma, f).$$

With a slight abuse of notation and terminology, let $\underline{\lambda}(x)$ denote the lower bound associated with market $x$, and refer to it as the market-specific bound. Applying Theorem 1 yields the following result.

**Corollary 4.** *Suppose that covariate distribution $f$ satisfies $f_s > \sup_{x\in\mathcal{X}} \underline{\lambda}(x)$ for every label $s$. Then $f \in WC(\mathcal{X})$. Conversely, suppose that $f \in WC(\mathcal{X})$. Then there exists some market $x \in \mathcal{X}$ such that $f_s > \underline{\lambda}(x)$ for all $s$.*

Lemma 1 implies that the maximum worst-case consumer surplus equals $\inf_{x\in\mathcal{X}} u^*(x)$, where $u^*(x)$ is the consumer surplus under the uniform monopoly price for market $x$. By Theorem 1, any $f$ satisfying $f_s > \sup_{x\in\mathcal{X}} \underline{\lambda}(x)$ guarantees $\inf_{\sigma\in\Sigma(f,x)} \underline{u}(\sigma, f) = u^*(x)$ for every $x \in \mathcal{X}$, and therefore $f \in WC(\mathcal{X})$. Conversely, if $f$ violates the lower bound in all markets of $\mathcal{X}$, then $\inf_{\sigma\in\Sigma(f,x)} \underline{u}(\sigma, f) < u^*(x)$ for each $x$, implying $f \notin WC(\mathcal{X})$.

Although satisfying the uniform lower bound for every market in $\mathcal{X}$ is sufficient for $f$ to achieve the maximum worst-case consumer surplus under ambiguity about $\mathcal{X}$, it is not necessary. The next example shows that a covariate distribution may belong to $WC(\mathcal{X})$ even if it violates the bound for some markets in $\mathcal{X}$.

**Example 2 (The sufficient condition in Corollary 4 is not necessary).** Let the set of valuations be $V = \{1, 2, 3\}$ and consider two possible markets $x = \left(\frac{9}{20}, \frac{1}{2}, \frac{1}{20}\right)$ and $x' = \left(\frac{1}{6}, \frac{1}{2}, \frac{1}{3}\right)$, so that the regulator faces ambiguity over $\mathcal{X} = \{x, x'\}$. In both markets the uniform monopoly price is 2, but the associated consumer surpluses differ: $u^*(x) = \frac{1}{20}$ and $u^*(x') = \frac{1}{3}$. The corresponding market-specific bounds are $\underline{\lambda}(x) = \frac{3}{20}$ and $\underline{\lambda}(x') = \frac{2}{3}$. Consider the covariate distribution $f = \left(\frac{1}{5}, \frac{4}{5}\right)$. This covariate distribution satisfies the market-specific bound for $x$ but not for $x'$. By Theorem 1, the worst-case consumer surplus equals $u^*(x)$ in the first market, whereas in the second market it is strictly below $u^*(x')$. However, for $x'$ the lowest attainable consumer surplus remains above $u^*(x)$. Indeed, the worst case for $x'$ arises from the feasible segmentation $\sigma$ in which, conditional on the first label, the monopolist perfectly learns that the consumer's valuation is 3, while all other consumers are pooled under the second label: $\sigma(\cdot \mid 1) = (0, 0, 1)$ and $\sigma(\cdot \mid 2) = \left(\frac{5}{24}, \frac{5}{8}, \frac{1}{6}\right)$. In this case, consumers with label 1 obtain zero surplus, while those with label 2 face price 2 and receive surplus $\frac{1}{6}$. Hence, $\underline{u}(\sigma, f) = \frac{4}{5} \times \frac{1}{6} = \frac{2}{15} > u^*(x) = \frac{1}{20}$, and the overall worst case across $\mathcal{X}$ is therefore determined by the first market. Thus, $f \in WC(\mathcal{X})$ despite violating the market-specific bound in one market.

The next example shows that the market-specific bound $\underline{\lambda}(x)$ can be arbitrarily small for some markets.

**Example 3 ($\underline{\lambda}$ can be arbitrarily small).** The set of consumers' valuations is given by $V = \{1, 2, 3\}$, and the market is $x_1 = \frac{2}{5}$, $x_2 = \frac{3}{5} - x_3$ and $x_3 \in \left(0, \frac{1}{10}\right]$. Figure 1 shows that $\underline{\lambda}$ decreases as $x_3$ decreases, and can become arbitrarily small. Note that the uniform monopoly price remains constant at 2 for all values of $x_3$. If a covariate distribution leads to consumer surplus below $u^*$, the monopolist must charge a price of 3 in some segment. For this to be feasible, it must be possible to pack at least a mass of $\frac{1}{3}$ consumers with valuation 3, which implies $f_s \leq 3x_3$.

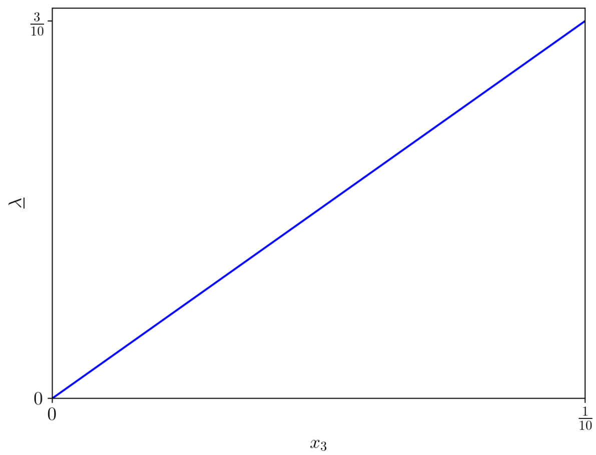


Figure 1: Value of $\underline{\lambda}$ for $x_1 = \frac{2}{5}$, $x_2 = \frac{3}{5} - x_3$ and $x_3 \in \left(0, \frac{1}{10}\right]$

Figure 1 shows that the market-specific bound is smaller in markets where demand at prices above the uniform monopoly price is lower, while demand at prices below or equal to the uniform monopoly price remains constant. Corollary 5 formalizes this idea and shows that the market-specific bound also decreases when demand at prices below or equal to the uniform monopoly price is higher.

**Corollary 5.** *Let $x, x' \in \Delta(V)$ be two markets, and suppose that the uniform monopoly price, $v_{i^*}$, is equal in both markets. Furthermore, assume that $\bar{x}_k \geq \bar{x}'_k$ for all valuations $v_k \leq v_{i^*}$, and $\bar{x}'_k \geq \bar{x}_k$ for all valuations $v_k > v_{i^*}$. Then $\underline{\lambda}(x) \leq \underline{\lambda}(x')$.*

*Proof.* For any $v_k > v_{i^*}$ and for any $v_j \leq v_{i^*}$ it is true that $v_k(\bar{x}'_k - \bar{x}_k) \geq 0 \geq v_j(\bar{x}'_j - \bar{x}_j)$. If I manipulate the later inequality, I get

$$\frac{v_k \bar{x}'_k}{v_j} + 1 - \bar{x}'_j \geq \frac{v_k \bar{x}_k}{v_j} + 1 - \bar{x}_j,$$

which leads to the desired result. $\square$

### 3.1 Dominance Order

After characterizing all worst-case optimal policies available to the regulator, a natural follow-up question arises: among the covariate distributions that satisfy the uniform lower bound $\underline{\lambda}$, which ones exhibit superior performance relative to others? Although every worst-case optimal policy guarantees a consumer surplus of at least $u^*$, it is natural to compare their performance beyond this worst-case guarantee.

To illustrate, consider the policy that only allows access to the trivial covariate distribution, i.e., $\mathcal{F} = \{f^\emptyset\}$. This policy resembles a ban on consumer information and is worst-case optimal. However, suppose that $f \in WC \setminus \{f^\emptyset\}$. By Lemma 1, $\underline{u}(\sigma, f) \geq u^*$ for every feasible segmentation $\sigma \in \Sigma(f)$. Moreover, if $\underline{u}(\sigma, f) > u^*$ for some feasible segmentation $\sigma \in \Sigma(f)$, then there is no reason for the regulator to withhold $f$ from the monopolist. Thus, a worst-case optimal policy may nevertheless be dominated by another worst-case optimal policy, making it natural to restrict attention to undominated policies.

Formally, for each covariate distribution $f$, let $U(f) = \{\underline{u}(\sigma, f) \mid \sigma \in \Sigma(f)\}$ denote the set of attainable consumer surplus payoffs. For sets $X, Y \subset \mathbb{R}$, write $X \succeq_I Y$ if $x \geq y$ for all $(x, y) \in X \times Y$. Then $f$ *dominates* $f'$ if $U(f) \succeq_I U(f')$ and $U(f') \not\succeq_I U(f)$. A covariate distribution is *dominated* if it is dominated by another covariate distribution, and *undominated* otherwise. Let

$$UD \equiv \left\{ f \mid f \in WC \text{ and there is no } f' \in WC \text{ that dominates } f \right\}.$$

The regulator is therefore interested in identifying the covariate distributions in $UD$. The following result provides a simple characterization of this set.

**Lemma 2.** *Let $F_2 \equiv \{f \in WC \mid \underline{u}(\sigma, f) > u^* \text{ for some } \sigma \in \Sigma(f)\}$ be the set of covariate distributions that never yield lower consumer surplus than $u^*$ and sometimes higher consumer surplus than $u^*$. Then $UD \equiv F_2$ if $F_2$ is non-empty, and $UD \equiv WC$ otherwise.*

*Proof.* Suppose that $F_2$ is non-empty. Pick any $f \in F_2$ and let $\sigma \in \Sigma(f)$ be the segmentation that yields a consumer surplus strictly higher than $u^*$. For any $f' \in WC$, $\sigma'(\cdot|s) = x^*$ for all $s$ is feasible. Thus, $\underline{u}(\sigma, f) > \underline{u}(\sigma', f') = u^*$; therefore, $f$ is undominated. Conversely, pick any $f \in UD$ and suppose that $f \notin F_2$. Then $\underline{u}(\sigma, f) = u^*$ for all $\sigma \in \Sigma(f)$; thus, any $f' \in F_2$ dominates $f$, which is a contradiction.

Now suppose that $F_2$ is empty. Then, for any $f \in WC$, we have that $\underline{u}(\sigma, f) = u^*$ for all $\sigma \in \Sigma(f)$, which implies that any $f$ is undominated, and therefore $UD \equiv WC$. □

The second main result of this paper fully characterizes the set $UD$.

**Theorem 2.** *Suppose that the covariate distribution $f$ achieves the maximum worst-case consumer surplus. Then $f$ yields a consumer surplus greater than $u^*$ for some feasible segmentation $\sigma \in \Sigma(f)$ if and only if $f_s < \bar{\lambda}$ for some label $s$, where*

$$\bar{\lambda} = \max_{v_j < v_{i^*}} 1 - \frac{v_{i^*}\bar{x}_{i*} - v_j \bar{x}^*_j}{v_{i^*} - v_j}.$$

*Only if direction of Theorem 2.* Suppose that $f \in WC$ and $\underline{u}(\sigma, f) > u^*$ for some $\sigma \in \Sigma(f)$. Then a segment $s$ must exist where the monopolist sets a price lower than $v_{i^*}$. Denote such a price by $v_{k_s}$. Since $v_{k_s}$ maximizes the monopolist's profits in segment $s$, and the monopolist breaks ties against consumers, it follows that

$$\begin{aligned}
\frac{1}{v_{i^*} - v_{k_s}} \underbrace{\left( v_{k_s} \sum_{j=k_s}^{K} \sigma(j|s) - v_{i^*} \sum_{j=i^*}^{K} \sigma(j|s) \right)}_{\text{Profits from price } v_{k_s} - \text{Profits from price } v_{i^*}} &= \frac{v_{k_s}}{v_{i^*} - v_{k_s}} \sum_{j=k_s}^{i^*-1} \sigma(j|s) + \sum_{j=1}^{i^*-1} \sigma(j|s) - 1 \\
&> 0. \qquad (5)
\end{aligned}$$

Furthermore, since $\sigma \in \Sigma(f)$, $\sigma(\cdot|s) f_s \leq x^*$. Then, if I multiply both sides of inequality 5 by $f_s$ I get that

$$\begin{aligned}
1 - \frac{v_{i^*}\bar{x}_{i*} - v_{k_s}\bar{x}^*_{k_s}}{v_{i^*} - v_{k_s}} &= \frac{v_{k_s}}{v_{i^*} - v_{k_s}} \sum_{j=k_s}^{i^*-1} x^*_j + \sum_{j=1}^{i^*-1} x^*_j \\
&\geq \left[ \frac{v_{k_s}}{v_{i^*} - v_{k_s}} \sum_{j=k_s}^{i^*-1} \sigma(j|s) + \sum_{j=1}^{i^*-1} \sigma(j|s) \right] f_s \\
&> f_s. \qquad (6)
\end{aligned}$$

Inequality 6 holds for some $v_{k_s} < v_{i^*}$. Hence,

$$f_s < \max_{v_j < v_{i^*}} 1 - \frac{v_{i^*}\bar{x}_{i*} - v_k \bar{x}_j^*}{v_{i^*} - v_j} = \bar{\lambda}.$$

$\square$

The proof of the necessity part of Theorem 2 shows that if a covariate distribution achieves consumer surplus strictly above $u^*$ under some feasible segmentation, then there must exist a segment in which the monopolist charges a price below the uniform monopoly price. For this to be optimal, the segment must place sufficient weight on consumers with valuations below the uniform monopoly price. Feasibility then requires that the corresponding label not be too prevalent in the population; otherwise, the segment would need to contain more low-valuation consumers than are available.

I defer the proof of the if part of Theorem 2 to Appendix B and provide a brief sketch here. The goal is to construct a feasible segmentation that yields a consumer surplus greater than $u^*$ for any $f$ that satisfies $f_s < \bar{\lambda}$ for some $s$. Since $f \in WC$, the process is straightforward: in the segment $s$ where $f_s < \bar{\lambda}$, pack as many consumers as needed with valuations lower than $v_{i^*}$. Ensure that the price is less than $v_{i^*}$ while maintaining a positive mass of consumers who enjoy a positive surplus. The condition $f_s < \bar{\lambda}$ ensures the feasibility of such a segment. Additionally, by Corollary 2, the prices in the other segments must be lower or equal to $v_{i^*}$. Therefore, consumer surplus must exceed $u^*$.

The following corollary provides sufficient and necessary conditions under which the set $F_2$ non-empty.

**Corollary 6.** *The set of covariate distributions $F_2$ is non-empty if and only if the uniform monopoly price, $v_{i^*}$, satisfies $v_{i^*} \notin \arg\min_{v_j \le v_{i^*}} \frac{\max_{v_i > v_{i^*}} v_i \bar{x}_i^*}{v_j} + 1 - \bar{x}_j^*$ and $\underline{\lambda} < \frac{1}{2}$.*

First, we require $\underline{\lambda} < \frac{1}{2}$; otherwise, by Corollary 1, the set $WC \setminus \{f^{\emptyset}\}$ is empty. Next, $F_2$ is non-empty if and only if $\bar{\lambda} > \underline{\lambda}$. This occurs when a valuation strictly below the uniform monopoly price minimizes the expression defining $\underline{\lambda}$. Intuitively, high demand at that valuation makes it easier to sustain a segment in which the monopolist optimally charges a lower price without violating feasibility, thereby creating room for covariate distributions that belong to $WC$ and achieve consumer surplus strictly above $u^*$.

**Corollary 7.** *Suppose that the covariate distribution $f$ belongs to $F_2$. Then there is a feasible segmentation $\sigma \in \Sigma(f)$ that yields both a consumer surplus and a producer surplus higher than those obtained through uniform pricing.*

Corollary 7 follows from the fact that if $f \in F_2$, then under some feasible segmentation the monopolist charges a price strictly below the uniform monopoly price in at least one segment. By the tie-breaking rule, this implies strictly higher profits in that segment than under the uniform monopoly price, while optimality implies weakly higher profits in all other segments. Summing across segments yields the result.

Figure 2 illustrates Corollary 7. The figure shows the surplus triangle derived by Bergemann et al. (2015). That is, all combinations of producer and consumer surplus that result from third-degree price discrimination. The point $(\pi^*, u^*)$ corresponds to the producer and consumer surplus achieved with uniform pricing. The shaded area in blue represents the market outcomes, achievable with third-degree price discrimination, that lead to increases in producer and consumer surplus. The covariate distributions characterized by Theorem 2 always achieve a point inside this blue region. Furthermore, for some feasible segmentation, they achieve the interior of the blue area.

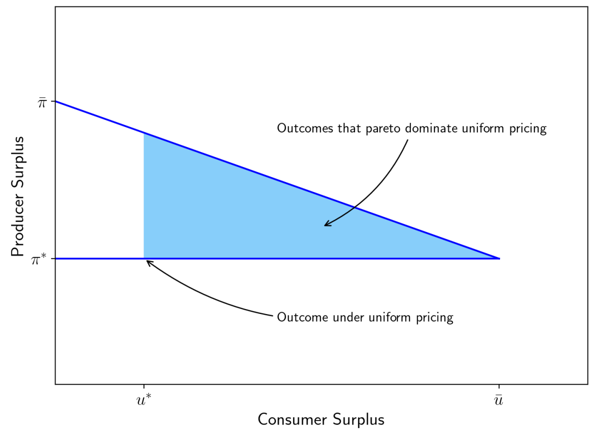


Figure 2: Surplus triangle

# 4 Extensions

In some markets, regulators might prioritize consumers but still care about the potential impact of consumer data on producer surplus. In addressing this concern, I examine the model when the regulator's objective is to maximize a weighted average of consumer and producer surplus, where the weight assigned to consumer surplus is weakly higher than the one assigned to producer surplus. For this purpose, denote by $\pi^*$ the producer surplus achieved with the uniform monopoly price $v_{i^*}$, and by $w^*_\alpha$ the $\alpha$-weighted surplus, where $\alpha \in \left[\frac{1}{2}, 1\right]$ is the weight assigned to $u^*$: $\pi^* = v_{i^*} \sum_{j=i^*}^{K} x^*_j$ and $w^*_\alpha = \alpha u^* + (1-\alpha)\pi^*$. As a special case, see that when $\alpha = \frac{1}{2}$, the regulator aims to maximize the total surplus, given by $w^*_{\frac{1}{2}} = \frac{1}{2} \sum_{j=i^*}^{K} v_j x^*_j$.

Additionally, let $\underline{\pi}(\sigma, f)$ and $\underline{w}_\alpha(\sigma, f)$ denote the producer surplus and the $\alpha$-weighted surplus, respectively, obtained when the monopolist has access to $f$ and he segments the market according to $\sigma$,

$$\underline{\pi}(\sigma, f) = \sum_{s=1}^{n} f_s v_{i^*_s} \sum_{j=i^*_s}^{K} \sigma(j|s), \quad \underline{w}_\alpha(\sigma, f) = \alpha \underline{u}(\sigma, f) + (1-\alpha)\underline{\pi}(\sigma, f),$$

where $v_{i^*_s}$ is the valuation defined in equation 1. Then, the set of covariate distributions that achieve the maximum worst-case $\alpha$-weighted surplus is given by

$$WC_\alpha \equiv \arg\max_{f \in F} \inf_{\sigma \in \Sigma(f)} \underline{w}_\alpha(\sigma, f).$$

The next result resembles Lemma 1 in the context where the regulator's objective is the $\alpha$-weighted surplus.

**Lemma 3.** *A covariate distribution $f$ achieves the maximum worst-case $\alpha$-weighted surplus if and only if $\underline{w}_\alpha(\sigma, f) \geq w^*_\alpha$ for all feasible segmentations $\sigma \in \Sigma(f)$.*

I omit the proof of Lemma 3, as it is analogous to the proof of Lemma 1. For any covariate distribution, the independent segmentation is feasible and yields an $\alpha$-weighted surplus of $w^*_\alpha$, implying that $w^*_\alpha$ is an upper bound on the worst-case $\alpha$-weighted surplus. Since the trivial covariate distribution attains this bound, the result follows.

Then, using Lemma 3, I can show that any $f$ that attains the maximum worst-case $\alpha$-weighted surplus must satisfy the uniform lower bound defined in Theorem 1.

**Theorem 3.** *Suppose that the covariate distribution $f$ belongs to $WC_\alpha$. Then $f_s > \underline{\lambda}$ for every label $s$.*

*Proof.* Towards a contradiction, suppose that $f \in WC_\alpha$ but $f_s \leq \underline{\lambda}$ for some label $s$. Then, the segmentation $\sigma$ constructed in the proof of the only if part of Theorem 1 is feasible given the marginals $f$ and $x^*$. Recall that the price charged by the monopolist in segment $s$ is higher than the uniform monopoly price $v_{i^*}$, and in all other segments, the price is no lower than $v_{i^*}$. Furthermore, segment $s$ contains a positive mass of consumers with a valuation equal to $v_{i^*}$. Hence, $\underline{w}_{\frac{1}{2}}(\sigma, f) < w^*_{\frac{1}{2}}$. Indeed, no consumer with a valuation lower than $v_{i^*}$ gets the good. For all valuations higher or equal to $v_{i^*}$, the mass of consumers that get the good is strictly lower than that of consumers that get the good with the uniform monopoly price.

If the $\frac{1}{2}$-weighted surplus decreases, then

$$u^* - \underline{u}(\sigma, f) > \underline{\pi}(\sigma, f) - \pi^*.$$

If I multiply the left-hand side of the inequality by $\alpha \in \left[\frac{1}{2}, 1\right]$ and the right-hand side by $1-\alpha$, the inequality is preserved. Thus, $\underline{w}_\alpha(\sigma, f) < w^*_\alpha$, a contradiction. □

The proof of Theorem 3 also shows where the assumption $\alpha \geq \frac{1}{2}$ is used. If a covariate distribution $f$ violates the bound $\underline{\lambda}$, the adversarial segmentation constructed in the proof of Theorem 1 causes a loss in consumer surplus, relative to uniform pricing, that exceeds any gain in producer surplus. Hence, any weighted surplus that assigns at least as much weight to consumer surplus as to producer surplus must also decrease.

Theorem 1 and Theorem 3 imply the next corollary.

**Corollary 8.** *The set of covariates that yield the maximum worst-case $\alpha$-weighted surplus is equivalent to the set of covariates that yield the maximum worst-case consumer surplus.*

The conclusion follows from initially observing, by Theorem 1, that $f \in WC$ if and only if $f_s > \underline{\lambda}$ for every label $s$. Subsequently, Theorem 3 implies that any $f \in WC_\alpha$ also belongs to $WC$. To establish the reverse inclusion, note that if $f \in WC$, it also achieves a consumer surplus weakly greater than $u^*$ for all feasible segmentations. Moreover, under third-degree price discrimination, the producer surplus is always weakly greater than that achieved with uniform pricing. Consequently, $f$ attains an $\alpha$-weighted surplus weakly greater than $w^*_\alpha$ for all feasible segmentations, and thus $f \in WC_\alpha$.

Finally, I will show that the set of undominated covariates that achieve the maximum worst-case $\alpha$-weighted surplus also remains the same. Let

$$UD_\alpha \equiv \left\{ f \mid f \in WC_\alpha \text{ and there is no } f' \in WC_\alpha \text{ that dominates } f \right\}.$$

Once more, the next result resembles Lemma 2 in the context where the regulator's objective is the $\alpha$-weighted surplus.

**Lemma 4.** *Let $F_\alpha \equiv \{f \in WC_\alpha \mid \underline{w}_\alpha(\sigma, f) > w^*_\alpha$ for some $\sigma \in \Sigma(f)\}$ be the set of covariates that never yield lower $\alpha$-weighted surplus than $w^*_\alpha$ and sometimes higher $\alpha$-weighted surplus than $w^*_\alpha$. Then $UD_\alpha \equiv F_\alpha$ if $F_\alpha$ is non-empty, and $UD_\alpha \equiv WC_\alpha$ otherwise.*

I omit the proof of Lemma 4, as it is identical to the proof of Lemma 2. If $f \in F_\alpha$, then $f$ cannot be dominated by any other covariate distribution $f'$. Indeed, for every $f'$ there exists a feasible segmentation $\sigma$, in which labels are independent of valuations, such that $\underline{w}^*_\alpha(\sigma, f') = w^*_\alpha$. Since $f \in F_\alpha$, there also exists a feasible segmentation $\sigma'$ such that $\underline{w}^*_\alpha(\sigma', f) > w^*_\alpha$.

Furthermore, the next result shows that is still true that any $f$ that never yields lower $\alpha$-weighted surplus than $w^*_\alpha$ and sometimes higher $\alpha$-weighted surplus than $w^*_\alpha$ must attach a mass lower than $\bar{\lambda}$ to some label.

**Theorem 4.** *Suppose that the covariate distribution $f$ belongs to $F_\alpha$. Then $f_s < \bar{\lambda}$ for some label $s$.*

*Proof.* Suppose that $f \in WC_\alpha$ and $\underline{w}_\alpha(\sigma, f) > w^*_\alpha$ for some $\sigma \in \Sigma(f)$. For this inequality to hold, there must be a segment where the price the monopolist charges differs from $v_{i^*}$. Additionally, by Corollary 2, in every segment, the price charged by the monopolist is weakly lower than $v_{i^*}$. Then, for some segment $s$, the price must be strictly lower than $v_{i^*}$. Moreover, if I follow the argument used to show the only if part of Theorem 2 I will get that whenever a price lower than $v_{i^*}$ is an optimal price for segment $s$, then $f_s < \bar{\lambda}$. $\square$

Intuitively, since every $f \in WC_\alpha$ satisfies the bound $\underline{\lambda}$, the monopolist never charges a price above the uniform monopoly price under any feasible segmentation. Hence, for the $\alpha$-weighted surplus to exceed its uniform-pricing level, some segment must be assigned a strictly lower price. By Theorem 2, this occurs if and only if $f_s < \bar{\lambda}$ for some label $s$.

Theorem 2 and Theorem 4 yield the next corollary

**Corollary 9.** *The set of covariates that never yield lower $\alpha$-weighted surplus than $w^*_\alpha$ and sometimes higher $\alpha$-weighted surplus than $w^*_\alpha$ is equivalent to the set of covariates that never yield lower consumer surplus than $u^*$ and sometimes higher consumer surplus than $u^*$.*

The assertion that any covariate that belongs to $F_\alpha$ is also part of the set $UD$ can be derived from Theorem 2 and Theorem 4. The reverse inclusion is established through a similar argument as delineated in Corollary 8: If $f$ never achieves a lower producer surplus than the one attained with uniform pricing, and, additionally, $f$ achieves a higher consumer surplus than $u^*$ for some segmentation, then it also attains a higher $\alpha$-weighted surplus for that specific segmentation than $w^*_\alpha$.

# 5 Conclusion

In this paper, I analyzed a setting where a regulator determines what data a firm can access to maximize consumer surplus. Furthermore, I assumed this regulator had limited knowledge of the relevant environment. Notably, she possesses non-Bayesian uncertainty regarding the correlation between data and consumers' willingness to pay.

Throughout this paper, I assumed that the monopolist uses consumer data for price discrimination. However, a promising direction for future work is to explore how the policies the regulator implements change when a firm uses the data for multiple purposes besides price discrimination. For instance, firms may use consumer data to offer consumers personalized products that better match their preferences. In this setting, it is still reasonable to assume that regulators have limited knowledge of the products a firm will recommend and the prices consumers will face. In particular, this will depend highly on the correlation between the data and willingness to pay. Therefore, the modeling ideas used in this paper can be helpful in such a setting: The regulator evaluates a covariate across all feasible joint distributions of consumers' characteristics and valuations for each product.

# A Proof of the Only If Direction of Theorem 1

## A.1 Greedy Algorithm to Construct Extreme Segmentations

The proof of the only if part of Theorem 1 builds on the algorithm proposed by Bergemann et al. (2015) that segments $x^*$ by means of a "greedy" procedure and creates segments which are extreme markets.

Before formally presenting the algorithm, I define an *extreme market*. Let $\mathcal{V}$ represent the set of non-empty subsets of $V = \{v_1, ..., v_K\}$. For every subset $S \in \mathcal{V}$, Bergemann et al. (2015) refer to $x^S \in \Delta(V)$ as an *extreme market* if it satisfies the properties that

1. no consumer has a valuation outside the set $S$, and

2. the monopolist is indifferent between charging any price inside the set $S$.

Bergemann et al. (2015) show that writing $\min S$ and $\max S$ for the smallest and greatest element of $S$, respectively, and denoting by $\mu(v_i, S)$ the smallest element of $S$ which is strictly greater than $v_i$, we must have

$$x_i^S = \begin{cases} 0 & \text{if } v_i \notin S; \\ \min S \left( \frac{1}{v_i} - \frac{1}{\mu(v_i,S)} \right) & \text{if } v_i \in S \text{ and } v_i \neq \max S; \\ \frac{\min S}{\max S} & \text{if } v_i = \max S. \end{cases} \tag{7}$$

Having defined $x^S$, I can now describe the steps of the greedy algorithm exactly as outlined by Bergemann et al. (2015): At the end of the iteration $l \geq 0$, the "residual" market of valuations not yet assigned to a segment is $x^l$, with $x^0 = x^*$, and the support of this residual is defined to be $S_l = \operatorname{supp} x^l$. I now describe what happens at iteration $l$, taking as inputs $\left(x^{l-1}, S_{l-1}\right)$. If $x^{l-1} = x^{S_{l-1}}$, then let $\alpha^l = 1$ and $x^l = 0$. Otherwise, define the projection $z(t)$ by

$$z(t) = x^{S_{l-1}} + t \cdot \left( x^{l-1} - x^{S_{l-1}} \right). \tag{8}$$

For this projection, let $\hat{t} = \inf\{t \geq 0 | z_i(t) < 0 \text{ for some } i\}$, such that $z_i\left(\hat{t}\right) \geq 0$ for all $i$ and $z_i\left(\hat{t}\right) = 0$ for some $i$. See that $\hat{t}$ is well defined as there is some $i$ for which $z_i(t)$ eventually hits zero. That is, there is some $i$ for which $x_i^{l-1} < x_i^{S_{l-1}}$. Indeed, since $x^{l-1} \neq x^{S_{l-1}}$, having $x_i^{l-1} \geq x_i^{S_{l-1}}$ for all $i$ would violate probabilities in $x^{l-1}$ summing to one. Furthermore, as all elements of $x^{l-1}$, and all elements of $x^{S_{l-1}}$, add up to one, it follows that $\sum_{j=1}^{K} z_j\left(\hat{t}\right) = 1$. Therefore, $z\left(\hat{t}\right) \in \Delta(V)$. The next residual market $x^l$ is defined to be $z\left(\hat{t}\right)$, and $\alpha^l$ is defined by

$$x^{l-1} = \alpha^l \cdot x^{S_{l-1}} + (1 - \alpha^l) \cdot x^l.$$

Now inductively define $S_l = \operatorname{supp} x^l$, which is a strict subset of $S_{l-1}$. The inductive hypothesis is that

$$x^* = \sum_{j=1}^{l} \alpha^j \prod_{i=1}^{j-1} (1 - \alpha^i) \cdot x^{S_{j-1}} + \prod_{i=1}^{l} (1 - \alpha^i) \cdot x^l, \tag{9}$$

meaning that the aggregate market is segmented by the $x^{S_j}$ extreme markets for $j < l$ and the residual market. This property is trivially satisfied for the base case $l = 1$ (with the convention that the empty product is equal to 1), and the choice of $\alpha^l$ guarantees that if the inductive hypothesis holds at $l - 1$, it will continue to hold at $l$ as well. The algorithm terminates at iteration $L + 1$

when $x^L = x^{S_L}$, which certainly has to be the case when $|S_L|= 1$. Therefore, $x^*$ is segmented by the extreme markets $\{x^{S_l}\}_{l=0}^{L}$ and the mass of consumers assigned to each segment is $y\left(x^{S_l}\right) = \alpha^{l+1}\prod_{j=1}^{l}(1-\alpha^j)$.

### A.2 Notation

Let $v_{\bar{i}} = \max\left\{\arg\max_{v_j > v_{i^*}}\ v_j \bar{x}_j^*\right\}$ and $v_{\underline{i}} = \min\left\{\arg\min_{v_j \leq v_{i^*}} \frac{v_{\bar{i}}\bar{x}_{\bar{i}}^*}{v_j} + 1 - \bar{x}_j^*\right\}$. Therefore, $\underline{\lambda} = \frac{v_{\bar{i}}\bar{x}_{\bar{i}}^*}{v_{\underline{i}}} + 1 - \bar{x}_{\underline{i}}^*$.

### A.3 Outline of the Proof

- **Step 1:** In the first step, I show that if $x^*$ is segmented using the greedy algorithm, then, if at some iteration there is a positive mass to assign of consumers with valuations higher than $v_{\bar{i}}$, there must also be a positive mass to assign of consumers with valuation $v_{\bar{i}}$. That is, if $x_i^l > 0$ for some $v_i > v_{\bar{i}}$, then $x_{\bar{i}}^l > 0$. Similarly, if there is a positive mass to assign of consumers with valuation $v_{\bar{i}}$, then there is a positive mass to assign of consumers with valuation $v_{\underline{i}}$; i.e., if $x_{\bar{i}}^l > 0$, then $x_{\underline{i}}^l > 0$ . Finally, at some point, the algorithm will run out of mass of consumers with valuations lower than $v_{\underline{i}}$, but there will be a positive mass of consumers with valuation $v_{\bar{i}}$; i.e., if $x_i^{l-1} > 0$ for some $v_i < v_{\underline{i}}$, then $x_{\bar{i}}^l > 0$.

- **Step 2:** Then, I use **Step 1** to show that the algorithm assigns a total mass of consumers equal to $\underline{\lambda}$ to all extreme markets that contain $v_{\bar{i}}$ in its support.

- **Step 3:** Finally, I use **Step 2** to show that if $f_s \leq \underline{\lambda}$ I can construct a feasible segmentation that leads to a consumer surplus strictly lower than $u^*$.

### A.4 Step 1

**Lemma 5.** *Suppose that $x_i^l > 0$ for some $v_i > v_{\bar{i}}$. Then $x_{\bar{i}}^l > 0$.*

*Proof of Lemma 5.* The inductive hypothesis is that $x_{\bar{i}}^{l-1} > 0$ and $v_{\bar{i}}\bar{x}_{\bar{i}}^{l-1} > v_j\bar{x}_j^{l-1}$ for all $v_j \in S_{l-1} \setminus \{v_1, v_2, ..., v_{\bar{i}}\}$. I will use this inductive hypothesis to show that if $x_i^l > 0$ for some $v_i > v_{\bar{i}}$, then $x_{\bar{i}}^l > 0$ and $v_{\bar{i}}\bar{x}_{\bar{i}}^l > v_j\bar{x}_j^l$ for all $v_j \in S_l \setminus \{v_1, v_2, ..., v_{\bar{i}}\}$.

Since $x^l \neq 0$, $x^l$ is defined by the projection $z(t)$ (equation 8) evaluated at $\hat{t} > 0$. Therefore, for any $v_j \in S_{l-1}$, $v_j\bar{x}_j^l$ is given by

$$\begin{aligned} v_j\bar{x}_j^l &= \hat{t}v_j\bar{x}_j^{l-1} - \left(\hat{t}-1\right)v_j\bar{x}_j^{S_{l-1}} \\ &= \hat{t}v_j\bar{x}_j^{l-1} - \left(\hat{t}-1\right)\min S_{l-1}, \end{aligned} \tag{10}$$

where for the second equality, I used the analytical expression for extreme markets (equation 7). Note that, by the inductive hypothesis, $\hat{t}v_{\bar{i}}\bar{x}_{\bar{i}}^{l-1} > \hat{t}v_j\bar{x}_j^{l-1}$ for all $v_j \in S_{l-1}\setminus\{v_1, v_2, ..., v_{\bar{i}}\}$. Additionally, the second term of equation 10 is the same for all valuations in $S_{l-1}$. Therefore, $v_{\bar{i}}\bar{x}_{\bar{i}}^l > v_j\bar{x}_j^l$ for all $v_j \in S_{l-1} \setminus \{v_1, v_2, ..., v_{\bar{i}}\}$, and, as $S_l \subset S_{l-1}$, the inequality is also true for all for all $v_j \in S_l \setminus \{v_1, v_2, ..., v_{\bar{i}}\}$. In particular, $v_{\bar{i}}\bar{x}_{\bar{i}}^l > v_k\bar{x}_k^l$, where $v_k$ is the smallest valuation strictly higher than $v_{\bar{i}}$ in $S_l$. See that $v_k$ exists as I assumed $x_i^l > 0$ for some $v_i > v_{\bar{i}}$. Moreover, since $v_k > v_{\bar{i}}$, it follows that $v_k\bar{x}_{\bar{i}}^l > v_k\bar{x}_k^l$, which leads to $\bar{x}_{\bar{i}}^l - \bar{x}_k^l = x_{\bar{i}}^l > 0$.

Finally, the base case satisfies that $x_{\bar{i}}^0 > 0$ and $v_{\bar{i}}\bar{x}_{\bar{i}}^0 > v_j\bar{x}_j^0$ for all $v_j \in S_0 \setminus \{v_1, v_2, ..., v_{\bar{i}}\}$, where $x^0 = x^*$. Thus, I can conclude that if $x_i^l > 0$ for some $v_i > v_{\bar{i}}$, then $x_{\bar{i}}^l > 0$. $\square$

**Lemma 6.** *Suppose that $x^l_{\bar{i}} > 0$. Then $x^l_{\underline{i}} > 0$.*

*Proof of Lemma 6.* The inductive hypothesis is that $x^{l-1}_{\underline{i}} > 0$ and

$$v_{\underline{i}} = \min \left\{ \arg \min_{v_j \in S_{l-1}} \frac{v_{\bar{i}} \bar{x}^{l-1}_{\bar{i}}}{v_j} + 1 - \bar{x}^{l-1}_j \right\}.$$

I will show that if $x^l_{\bar{i}} > 0$, then $x^l_{\underline{i}} > 0$ and $v_{\underline{i}} = \min \left\{ \arg\min_{v_j \in S_l} \frac{v_{\bar{i}} \bar{x}^l_{\bar{i}}}{v_j} + 1 - \bar{x}^l_j \right\}$.

If I use the analytical expression for extreme markets (equation 7), it is easy to show that, for all $v_j \in S_{l-1}$,

$$\frac{v_{\bar{i}} \bar{x}^{S_{l-1}}_{\bar{i}}}{v_j} + 1 - \bar{x}^{S_{l-1}}_j = \frac{v_{\bar{i}}}{v_j} \frac{\min S_{l-1}}{v_{\bar{i}}} + 1 - \frac{\min S_{l-1}}{v_j} = 1.$$

Furthermore, since $x^l \neq 0$, $x^l$ is defined by the projection $z(t)$ (equation 8) evaluated at $\hat{t} > 0$. Therefore,

$$\frac{v_{\bar{i}} \bar{x}^l_{\bar{i}}}{v_j} + 1 - \bar{x}^l_j = \hat{t} \left( \frac{v_{\bar{i}} \bar{x}^{l-1}_{\bar{i}}}{v_j} + 1 - \bar{x}^{l-1}_j \right) - (\hat{t} - 1) \left( \frac{v_{\bar{i}} \bar{x}^{S_{l-1}}_{\bar{i}}}{v_j} + 1 - \bar{x}^{S_{l-1}}_j \right). \tag{11}$$

Note that, by the inductive hypothesis, $v_{\underline{i}}$ is the smallest element in $S_{l-1}$ to minimize the first term in parentheses, and before I showed that the second term in parenthesis is equal to 1 for all $v_j \in S_{l-1}$. Then, $v_{\underline{i}}$ is the smallest element in $S_{l-1}$ that minimizes 11. Furthermore, as $S_l \subset S_{l-1}$, the value of 11 is strictly lower under $v_{\underline{i}}$ compared to any lower valuation in $S_l$, and weakly lower compared to any higher valuation in $S_l$. In particular, denote by $v_k$ the smallest element in $S_l$ which is strictly greater than $v_{\underline{i}}$, where $v_k$ exists as $x^l_{\bar{i}} > 0$ and $v_{\bar{i}} > v_{\underline{i}}$. Then, see that

$$\begin{aligned} \frac{v_{\bar{i}} \bar{x}^l_{\bar{i}}}{v_k} + 1 - \bar{x}^l_k &\geq \frac{v_{\bar{i}} \bar{x}^l_{\bar{i}}}{v_{\underline{i}}} + 1 - \bar{x}^l_{\underline{i}} \\ x^l_{\underline{i}} = \bar{x}^l_{\underline{i}} - \bar{x}^l_k &\geq v_{\bar{i}} \bar{x}^l_{\bar{i}} \left( \frac{1}{v_{\underline{i}}} - \frac{1}{v_k} \right). \end{aligned} \tag{12}$$

Since $x^l_{\bar{i}} > 0$, the left-hand side of 12 is strictly positive, therefore, $x^l_{\underline{i}} > 0$.

Finally, the base case satisfies that $v_{\underline{i}} = \min \left\{ \arg\min_{v_j \leq v_{i^*}} \frac{v_{\bar{i}} x^0_{\bar{i}}}{v_j} + 1 - \bar{x}^0_j \right\}$ and $x^0_{\underline{i}} > 0$, where $x^0 = x^*$. The only issue is that $v_{\underline{i}}$ is the smallest valuation in the $\arg\min$ when the $\min$ is taken across all valuations lower or equal than $v_{i^*}$, and I need the $\min$ to be across all valuations in $S_0 = \operatorname{supp} x^*$. However, $v_{\underline{i}}$ is still the smallest valuation in the $\arg\min$ when the $\min$ is taken across $S_0$. Indeed, towards a contradiction, suppose that $v_j > v_{i^*}$ and

$$\begin{aligned} \frac{v_{\bar{i}} \bar{x}^*_{\bar{i}}}{v_j} + 1 - \bar{x}^*_j &\leq \frac{v_{\bar{i}} \bar{x}^*_{\bar{i}}}{v_{i^*}} + 1 - \bar{x}^*_{i^*} \\ v_{i^*} v_j \bar{x}^*_{i^*} &\leq (v_j - v_{i^*}) v_{\bar{i}} \bar{x}^*_{\bar{i}} + v_{i^*} v_j \bar{x}^*_j \\ v_{i^*} v_j \bar{x}^*_{i^*} &< (v_j - v_{i^*}) v_{i^*} \bar{x}^*_{i^*} + v_{i^*} v_{i^*} \bar{x}^*_{i^*} \\ \bar{x}^*_{i^*} &< \bar{x}^*_{i^*}, \end{aligned}$$

where the strict inequality follows from the fact that $v_{i^*}$ is the highest optimal price; therefore, $v_{i^*}\bar{x}^*_{i^*} > v_j\bar{x}^*_j$. Then, $\frac{v_{\bar{i}}\bar{x}^*_{\bar{i}}}{v_j} + 1 - \bar{x}^*_j > \frac{v_{\bar{i}}\bar{x}^*_{\bar{i}}}{v_{i^*}} + 1 - \bar{x}^*_{i^*} \geq \frac{v_{\bar{i}}\bar{x}^*_{\bar{i}}}{v_{\underline{i}}} + 1 - \bar{x}^*_{\underline{i}}$. □

**Lemma 7.** *Suppose that $x_i^{l-1} > 0$ for some $v_i < v_{\underline{i}}$. Then $x_{\bar{i}}^l > 0$.*

*Proof of Lemma 7.* The inductive hypothesis is that $x_{\bar{i}}^{l-1} > 0$. I will show that if $x_i^{l-1} > 0$ for some $v_i < v_{\underline{i}}$, then $x_{\bar{i}}^l > 0$.

Since $x_{\bar{i}}^{l-1} > 0$, Lemma 6 implies that $x_{\underline{i}}^{l-1} > 0$ and

$$v_{\underline{i}} = \min\left\{\arg\min_{v_j \in S_{l-1}} \frac{v_{\bar{i}}\bar{x}_{\bar{i}}^{l-1}}{v_j} + 1 - \bar{x}_j^{l-1}\right\}. \tag{13}$$

Equation 13 together with the assumption that $x_i^{l-1} > 0$ for some $v_i < v_{\underline{i}}$ implies that $x^{l-1} \neq x^{S_{l-1}}$. Otherwise, there is a valuation $v_i < v_{\underline{i}}$ and $v_i \in \arg\min_{v_j \in S_{l-1}} \frac{v_{\bar{i}}\bar{x}_{\bar{i}}^{S_{l-1}}}{v_j} + 1 - \bar{x}_j^{S_{l-1}}$. Indeed, in the proof of Lemma 6 I showed the term inside the $\arg\min$ equals 1 for all $v_j \in S_{l-1}$. The fact that $x^{l-1} \neq x^{S_{l-1}}$ implies that $x^l \neq 0$, and therefore $x^l$ is defined by the projection $z(t)$ (equation 8) evaluated at $\hat{t} > 0$.

Denote by $v_k$ the greatest element in $S_{l-1}$ that it is strictly smaller than $v_{\underline{i}}$, where $v_k$ exists as I assumed that $x_i^{l-1} > 0$ for some $v_i < v_{\underline{i}}$. Equation 13 leads to

$$\begin{aligned} \frac{v_{\bar{i}}\bar{x}_{\bar{i}}^{l-1}}{v_k} + 1 - \bar{x}_k^{l-1} &> \frac{v_{\bar{i}}\bar{x}_{\bar{i}}^{l-1}}{v_{\underline{i}}} + 1 - \bar{x}_{\underline{i}}^{l-1} \\ v_{\bar{i}}\bar{x}_{\bar{i}}^{l-1} &> \left(\frac{1}{v_k} - \frac{1}{v_{\underline{i}}}\right)^{-1} x_k^{l-1}. \end{aligned} \tag{14}$$

Furthermore, the fact that $x^l = z\left(\hat{t}\right)$ implies that

$$\begin{aligned} x_k^l &= \hat{t}x_k^{l-1} - \left(\hat{t}-1\right)x_k^{S_{l-1}} \\ &= \hat{t}x_k^{l-1} - \left(\hat{t}-1\right)\min S_{l-1}\left(\frac{1}{v_k} - \frac{1}{v_{\underline{i}}}\right), \quad \text{and} \\ \bar{x}_{\bar{i}}^l &= \hat{t}\bar{x}_{\bar{i}}^{l-1} - \left(\hat{t}-1\right)\bar{x}_{\bar{i}}^{S_{l-1}} \\ &= \hat{t}\bar{x}_{\bar{i}}^{l-1} - \left(\hat{t}-1\right)\frac{\min S_{l-1}}{v_{\bar{i}}}. \end{aligned}$$

Since $x_k^l \geq 0$, $\hat{t}\left(\frac{1}{v_k} - \frac{1}{v_{\underline{i}}}\right)^{-1} x_k^{l-1} \geq \left(\hat{t}-1\right)\min S_{l-1}$, then inequality 14 implies that $\hat{t}v_{\bar{i}}\bar{x}_{\bar{i}}^{l-1} > \left(\hat{t}-1\right)\min S_{l-1}$, which leads to $\bar{x}_{\bar{i}}^l > 0$. Moreover, if $\bar{x}_{\bar{i}}^l > 0$, then $x_i^l > 0$ for some $v_i \geq v_{\bar{i}}$. If $v_i = v_{\bar{i}}$ I get the desired result. If $v_i > v_{\bar{i}}$, Lemma 5 implies that also $x_{\bar{i}}^l > 0$, which also gives me the desired result.

Finally, for the base case, it is true that $x_{\bar{i}}^0 > 0$. Therefore, I can conclude that if $x_i^{l-1} > 0$ for some $v_i < v_{\underline{i}}$, then $x_{\bar{i}}^l > 0$. □

### A.5 Step 2

**Lemma 8.** *Suppose that $T > 0$ is the first step at which $x_{\bar{i}}^T = 0$. Then $\sum_{j=0}^{T-1} y\left(x^{S_j}\right) = \underline{\lambda}$.*

*Proof of Lemma 8.* Denote by $\{l_1, l_2, ..., l_N\}$ the sub-sequence of steps lower than $T$ such that at step $l_n$, $\min S_{l_n} = \min \operatorname{supp} x^{l_n}$ becomes for the first time the minimum valuation. That is, $\min S_{l_n} \neq \min S_l$ for all $l < l_n$. Since $S_0 = \operatorname{supp} x^*$ and $S_l \subset S_{l-1}$, is immediate that $l_1 = 0$, $\min S_{l_1} = \min \operatorname{supp} x^*$ and $\min S_{l_1} < \min S_{l_2} < \cdots < \min S_{l_N}$. Additionally, by assumption, $x^l_{\bar{i}} > 0$ for all $l < T$, by Lemma 6, $x^l_{\underline{i}} > 0$ for all $l < T$, and, by Lemma 7, if $v_i < v_{\underline{i}}$, then $x^l_i = 0$ for some $l < T$. Therefore, $\min S_{l_N} = v_{\underline{i}}$ and, for all $j \in \{l_n, ..., l_{n+1} - 1\}$,

$$\begin{aligned} 1 - \bar{x}^{S_j}_{\underline{i}} &= \sum_{k=1}^{\underline{i}-1} x^{S_j}_k = \min S_{l_n} \left( \frac{1}{\min S_{l_n}} - \frac{1}{v_{\underline{i}}} \right) \\ &= 1 - \frac{\min S_{l_n}}{v_{\underline{i}}}, \quad \text{and} \qquad (15) \\ \bar{x}^{S_j}_{\bar{i}} &= \sum_{k=\bar{i}}^{K} x^{S_j}_k = \frac{\min S_{l_n}}{v_{\bar{i}}}, \qquad (16) \end{aligned}$$

where I am using the analytical expression for extreme markets (equation 7).

Next, see that if I evaluate equation 9 at step $T$ and I add it across all valuations strictly lower than $v_{\underline{i}}$ I get that

$$\begin{aligned} 1 - \bar{x}^*_{\underline{i}} = \sum_{k=1}^{\underline{i}-1} x^*_k &= \sum_{j=1}^{T} \alpha^j \prod_{i=1}^{j-1} (1-\alpha^i) \sum_{k=1}^{\underline{i}-1} x^{S_{j-1}}_k + \prod_{i=1}^{T} (1-\alpha^i) \sum_{k=1}^{\underline{i}-1} x^T_k \\ &= \sum_{n=1}^{N} \sum_{j=l_n+1}^{l_{n+1}} \alpha^j \prod_{i=1}^{j-1} (1-\alpha^i) \sum_{k=1}^{\underline{i}-1} x^{S_{j-1}}_k \\ &= \sum_{n=1}^{N} \sum_{j=l_n+1}^{l_{n+1}} \alpha^j \prod_{i=1}^{j-1} (1-\alpha^i) \left(1 - \frac{\min S_{l_n}}{v_{\underline{i}}}\right) \\ &= \sum_{n=1}^{N} \sum_{j=l_n+1}^{l_{n+1}} \alpha^j \prod_{i=1}^{j-1} (1-\alpha^i) - \sum_{n=1}^{N} \frac{\min S_{l_n}}{v_{\underline{i}}} \sum_{j=l_n+1}^{l_{n+1}} \alpha^j \prod_{i=1}^{j-1} (1-\alpha^i), \end{aligned}$$

where I am using the convention that $l_{N+1} = T$, in the second equality I used the assumption that $x^T_{\bar{i}} = 0$, hence Lemma 7 implies that $x^T_k = 0$ for all $v_k < v_{\underline{i}}$, and in the third equality I used equation 15. Furthermore, recall that the mass of consumers assigned to the extreme market $x^{S_j}$ is equal to $y\left(x^{S_j}\right) = \alpha^{j+1} \prod_{i=1}^{j} (1-\alpha^i)$. Thus,

$$\sum_{j=0}^{T-1} y\left(x^{S_j}\right) = 1 - \bar{x}^*_{\underline{i}} + \sum_{n=1}^{N} \frac{\min S_{l_n}}{v_{\underline{i}}} \sum_{j=l_n+1}^{l_{n+1}} \alpha^j \prod_{i=1}^{j-1} (1-\alpha^i). \qquad (17)$$

If I now evaluate equation 9 at step $T$ and I add it across all valuations greater or equal than $v_{\bar{i}}$

I get that

$$\bar{x}^*_{\bar{i}} = \sum_{k=\bar{i}}^{K} x^*_k = \sum_{n=1}^{N} \sum_{j=l_n+1}^{l_{n+1}} \alpha^j \prod_{i=1}^{j-1}(1-\alpha^i) \sum_{k=\bar{i}}^{K} x_k^{S_{j-1}} + \prod_{i=1}^{T}(1-\alpha^i) \sum_{k=\bar{i}}^{K} x^l_k$$
$$= \sum_{n=1}^{N} \frac{\min S_{l_n}}{v_{\bar{i}}} \sum_{j=l_n+1}^{l_{n+1}} \alpha^j \prod_{i=1}^{j-1}(1-\alpha^i), \tag{18}$$

where in the second equality I used equation 16, and the fact that, by Lemma 5, $x^T_k = 0$ for all $v_k \geq v_{\bar{i}}$. Finally, multiplying and dividing both sides of equation 18 by $v_{\bar{i}}$ and $v_{\underline{i}}$, respectively, and combining the resulting equation with equation 17 yields

$$\sum_{j=0}^{T-1} y\left(x^{S_j}\right) = \frac{v_{\bar{i}} \bar{x}^*_{\bar{i}}}{v_{\underline{i}}} + 1 - \bar{x}^*_{\underline{i}} = \underline{\lambda},$$

which is the desired result. □

### A.6 Step 3

*Only if direction of Theorem 1.* Suppose that $\underline{u}(\sigma, f) \geq u^*$ for all $\sigma \in \Sigma(f)$. Towards a contradiction, assume that $f_s \leq \underline{\lambda}$ for some label $s$. Let $T \geq 0$ be the smallest step at which the mass of consumers assigned to all extreme markets up to $T$, $\left\{x^{S_j}\right\}_{j=0}^{T}$, is greater or equal than $f_s$. That is, $\sum_{j=0}^{T-1} y\left(x^{S_j}\right) < f_s \leq \sum_{j=0}^{T} y\left(x^{S_j}\right)$ (with the convention that the empty sum is equal to 0). Then, define $\sigma(\cdot|s)$ as

$$\sigma(\cdot|s) = \sum_{j=0}^{T-1} \frac{y\left(x^{S_j}\right)}{f_s} x^{S_j} + \left(1 - \sum_{j=0}^{T-1} \frac{y\left(x^{S_j}\right)}{f_s}\right) x^{S_T}.$$

See that $\sigma(\cdot|s)$ is a weighted average of all extreme markets up to step $T$. Therefore, $\sigma(\cdot|s)$ is a well defined market; i.e., $\sigma(\cdot|s) \in \Delta(V)$. Furthermore, since $f_s \leq \underline{\lambda}$, Lemma 8 implies that $v_{\bar{i}} \in \operatorname{supp} x^{S_j}$ for all $j \leq T$. Therefore, $v_{\bar{i}}$ is an optimal price for all extreme markets up to step $T$, and thus $v_{\bar{i}}$ is an optimal price for segment $s$. Moreover, because the monopolist breaks ties against consumers, he must set a price weakly greater than $v_{\bar{i}}$ for segment $s$. Additionally, equation 9 ensures that $\sigma(\cdot|s) f_s \leq x^*$, with strict inequality for some $v_k \in V$. Hence, I am not assigning more mass of consumers than the total available for any valuation.

For any other label $s' \neq s$ set $\sigma(\cdot|s')$ equal to

$$\sigma(\cdot|s') = \frac{x^* - \sigma(\cdot|s) f_s}{1 - f_s}.$$

In other words, I am distributing evenly the remaining mass of consumers of each valuation across all labels different from $s$. Note that $\sigma$ is a feasible segmentation given the marginals $x^*$ and $f^*$. Moreover, the monopolist must set the same price for every segment $s' \neq s$, which is no lower than $v_{i^*}$. Indeed, suppose that the profits obtained from setting a price equal to $v_j$ are higher than charging $v_{i^*}$ and $v_j < v_{i^*}$. Then, $v_j \in \operatorname{supp} \sigma(\cdot|s')$, which is equivalent to $x^*_j > \sigma(j|s) f_s$; thus, $v_j \in \operatorname{supp} x^{S_l}$ for all $l \leq T$. However, this implies that $v_j$ is also an optimal price in segment $s$. Hence, $v_j$ leads to higher profits than $v_{i^*}$ for the aggregate market, a contradiction.

Since the consumers in segment $s$ pay a price higher than $v_{i^*}$ without counterbalancing benefits to consumers in the remaining segments, $\sigma$ leads to a consumer surplus lower than $u^*$ but this is a contradiction. $\square$

## B Proofs of results in Section 3

*If direction of Theorem 2.* Suppose that $f \in WC$ and $f_s < \bar{\lambda}$ for some label $s$. Then, define $\sigma(\cdot|s)$ as follows. Let $\sigma(j|s) = \frac{x_j^*}{f_s}$ for all $v_j < v_{i^*}$, and for all valuations $v_j \geq v_{i^*}$ set $\sigma(j|s)$ such that two conditions are satisfied:

1. $\sigma(j|s) \leq \frac{x_j^*}{f_s}$ and
2. $\sum_{j=i^*}^{K} \sigma(j|s) = \frac{\bar{x}_{i^*}^*}{f_s} + 1 - \frac{1}{f_s}$.

Some observations of the above segment. First, it is always possible to find numbers that satisfy conditions (1) and (2). Indeed, if I set $\sigma(j|s) = \frac{x_j^*}{f_s}$ for all $v_j \geq v_{i^*}$ I will get that $\sum_{j=i^*}^{K} \sigma(j|s) = \frac{\bar{x}_{i^*}^*}{f_s}$, which is greater than $\frac{\bar{x}_{i^*}^*}{f_s} + 1 - \frac{1}{f_s}$, then I can reduce $\sigma(j|s)$ for any $v_j$ until both terms are equal. Second, I am assigning the whole mass of consumers with a valuation lower than $v_{i^*}$, but I am not assigning the whole mass of consumers with a valuation weakly higher than $v_{i^*}$. Third, by construction $\sum_{j=1}^{K} \sigma(j|s) = 1$, however this does not imply that $\sigma(\cdot|s) \in \Delta(V)$, as it is possible that $\sum_{j=1}^{i^*-1} \frac{x_j^*}{f_s} > 1$. For now, assume that $\sum_{j=1}^{i^*-1} \frac{x_j^*}{f_s} < 1$ such that $\sigma(\cdot|s) \in \Delta(V)$.

Next, I want to show that the monopolist will charge a price lower than $v_{i^*}$ in segment $s$. Since $f \in WC$, Corollary 2 implies that, for all feasible segmentations, the price in every segment must be weakly lower than $v_{i^*}$. Then, it is enough to check that there exists some $v_k < v_{i^*}$ such that the profits from charging $v_k$ are higher than the profits from charging $v_{i^*}$. The profits from charging $v_k$ and $v_{i^*}$ are equal to

$$\begin{aligned}
v_k \sum_{j=k}^{K} \sigma(j|s) &= v_k \left( \sum_{j=k}^{i^*-1} \sigma(j|s) + \sum_{j=i^*}^{K} \sigma(j|s) \right) \\
&= v_k \left( \sum_{j=k}^{i^*-1} \frac{x_j^*}{f_s} + \frac{\bar{x}_{i^*}^*}{f_s} + 1 - \frac{1}{f_s} \right) \\
&= v_k \left( \frac{\bar{x}_k^*}{f_s} + 1 - \frac{1}{f_s} \right), \qquad (19) \\
v_{i^*} \sum_{j=i^*}^{K} \sigma(j|s) &= v_{i^*} \left( \frac{\bar{x}_{i^*}^*}{f_s} + 1 - \frac{1}{f_s} \right). \qquad (20)
\end{aligned}$$

Hence, it is enough to check that 19 is greater than 20 for some $v_k < v_{i^*}$,

$$\begin{aligned}
v_k \left( \frac{\bar{x}_k^*}{f_s} + 1 - \frac{1}{f_s} \right) &> v_{i^*} \left( \frac{\bar{x}_{i^*}^*}{f_s} + 1 - \frac{1}{f_s} \right) \\
\frac{v_{i^*} - v_k - (v_{i^*}\bar{x}_{i^*}^* - v_k\bar{x}_k^*)}{f_s} &> v_{i^*} - v_k \\
1 - \frac{v_{i^*}\bar{x}_{i^*}^* - v_k\bar{x}_k^*}{v_{i^*} - v_k} &> f_s. \qquad (21)
\end{aligned}$$

Inequality 21 holds for some $v_k < v_{i^*}$, since $f_s < \bar{\lambda} = \max_{v_j < v_{i^*}} \; 1 - \frac{v_{i^*}\bar{x}_{i*} - v_j \bar{x}^*_j}{v_{i^*} - v_j}$. Therefore, the monopolist charges a price lower than $v_{i^*}$. It remains to define $\sigma$ for the other labels $s' \neq s$. For this purpose, let $\{\sigma(\cdot|s')\}_{s \neq s'}$ be any collection of distributions that together with $\sigma(\cdot|s)$ is consistent with $f$ and $x^*$. These distributions exist because, for segment $s$, I am not assigning more mass of consumers than the total available for any valuation. Moreover, Corollary 2 implies that the price charged by the monopolist in every segment $s' \neq s$ is weakly lower than $v_{i^*}$. Then, all consumers with a valuation weakly higher than $v_{i^*}$ pay a price weakly lower than $v_{i^*}$, and, since I assumed that $\sum_{j=1}^{i^*-1} \frac{x^*_j}{f_s} < 1$, a positive mass of them pay a price strictly lower than $v_{i^*}$. Thus, consumer surplus increases relative to $u^*$.

If $\sum_{j=1}^{i^*-1} \frac{x^*_j}{f_s} \geq 1$ the result still holds. Particularly, I can now construct a $\sigma(\cdot|s)$ such that for a sufficiently small $\varepsilon \in (0,1)$ the following 3 conditions are satisfied:

1. $\sigma(j|s) \leq \frac{x^*_j}{f_s}$ for all $v_j \in V$,
2. $\sum_{j=1}^{i^*-1} \sigma(j|s) = 1 - \varepsilon$ and $\sum_{j=i^*}^{K} \sigma(j|s) = \varepsilon$, and
3. the monopolist charges a price lower than $v_{i^*}$ in segment $s$.

Then, using the same logic as above, one will get that for the other segments, the price must be weakly lower than $v_{i^*}$; therefore, consumer surplus increases relative to $u^*$. □

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